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MATHEMATICAL MODEL TO FIND THE HRV WITH GROUNDED & NON GROUNDED GROUPS USING BDP AND GAMMA DISTRIBUTION

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Abstract:

Over the last few years, the utilization of integrative biophysics for medical application has been increasing in popularity. Grounding or earthing is the oldest and most basic form of natural bioelectric potential that supports physiological and electrophysiological changes in the body. Since previous investigations have shown that grounding profoundly affects skin conductance within seconds, we hypothesized that grounding may also improve heart rate variability (HRV). In this paper we estimate the heart rate variability with grounded & non grounded groups using birth death process through gamma distribution.

Key Words: Heart Rate Variability (HRV), Stationary Process, Birth Death Process (BDP) & Gamma Distribution

1. Introduction:

Grounding or earthing is defined as placing one's bare feet on the ground, whether it be dirt, grass, sand, or concrete. It is known that the earth maintains a negative electrical potential on its surface [2 & 20]. When in direct contact with the ground, the earth's electrons are conducted to the human body, bringing it to the same electrical potential as the earth [3 & 9]. Living in direct contact with the earth grounds the body, inducing favorable physiological and electrophysiological changes that promote optimum health [10]. Regulation of circadian rhythms and improved sleep and nighttime cortisol dynamics reflect a few changes associated with favorable autonomic nervous system (ANS) function that can come about with grounding [5 & 10]. The many unpredictable sociological, economic, and political events of the 21st century have increased the stress of modern living as compared to earlier and simpler times. As a result, more and more people live day to day in unrelenting states of heightened physiological arousal. These physiological states involve chronic over activation of the ANS.

Situations that balance the over stressed sympathetic limb of the autonomic nervous system also support the parasympathetic nervous system (PNS) and result in a decrease in sympathetic tone and improved clinical outcomes for stress. Interventions such as exercise, supplementation with omega-3 essential fatty acids, and medicating with supportive pharmaceutical agents all support the ANS [8]. Hence, when choosing antihypertensive pharmaceutical therapy, it is important to consider the drug's action on the ANS.

Heart rate variability (HRV) refers to beat to beat alterations in heart rate. During resting conditions, the electrocardiogram (ECG) in normal individuals demonstrates periodic variation in R-R intervals. Such HRV measurements provide reliable, noninvasive information on the autonomic nervous system, including its vagal and sympathetic components. The Karlin-McGregor representation for the transition probabilities of a birth death process with an absorbing bottom state involves a sequence of orthogonal polynomials and the corresponding measure. This representation can be generalized to a setting in which a transition to the absorbing state is possible from any state rather than just one state. Measurements of HRV with grounded & non grounded groups were estimated by using birth death process through gamma distribution.

2. Quasi Stationary:

By a famous result of [11], the transition probabilities of a birth death process on the integers can, under suitable conditions, be expressed in terms of a sequence of orthogonal polynomials and their orthogonalizing measure. This representation has led to detailed knowledge of many specific birth death processes and to considerable insight into the behavior of birth death processes in general. Evidently, it is of interest to investigate to what extent properties of birth death processes retain their validity if one allows more general transition structures. Such investigations are usually hampered by the fact that the orthogonal polynomial representation for the transition probabilities and the analytical tools that go with it are no longer available. The class of processes which is the subject of this article and which comprises an outwardly mild generalization of birth death processes does not have this drawback. At the same time, the class is interesting because it displays several of the phenomena that occur beyond the setting of the pure birth death process.

Concretely, we will consider birth death processes on the set $\{-1,0,1,\dots\}$ with -1being an absorbing bottom state, and the additional feature that absorption in one stepmay occur from any state rather than just one state. In particular, the existenceand the shape of quasi-stationary distributions will be our main concern. It has recently been shown in [18] that an orthogonal polynomial representation for the transition probabilities remains valid in this setting, so that the orthogonal polynomial toolbox may be used to analyze the behavior of such a process. In fact, the existence of quasi stationary distributions will be shown to depend on the asymptotic behavior of the orthogonal polynomials involved. Quasi stationary for birth death processes with killing has recently been studied in a discrete time setting. In this setting, the analysis is simpler because the asymptotic behavior of the pertinent orthogonal polynomials plays a less restrictive role. A recent paper addresses related problems in the setting of diffusions with killing. Here we study the quasi stationary behavior of the processes at hand. Our results comprise a characterization of quasi stationary distributions for birth death processes with killing, and some sufficient conditions for their existence.

2.1. Definitions and General Results.

A quasi stationary distribution for $\mathcal{K}$is a properprobability distribution $m \coloneqq (m_j, j \in C)$, such that for all $t \geq 0$,

$$P_m\{X(t) = j \mid T > t\} = m_j, j \in C$$

That is, m_j is a Quasi stationary distribution if the state probabilities of $\mathcal{K}$at time t, conditionalon the chain being in Cat time t, do not vary with $t$when $m$is chosen as initialdistribution.We note that

$$P_m\{X(t) = j \mid T > t\} = \frac{P_m\{X(t)=j\}}{P_m\{T>t\}}$$

while $P_m\{X(t) = j\} \to 0$ as $t \to \infty$for all $j \in C$and any initial distribution m. So, $m$can be a Quasi stationary distribution only if $P_m\{T > t\} \to 0$ as $t \to \infty$, that is, if absorption iscertain, our assumption throughout this section.

It will be convenient to introduce another concept. Namely, a proper probability distribution $\{m_j, j \in C\}$ over the non absorbing states is called $x$invariant for Q (the q matrix of $\mathcal{K}$) for some real x if

$$\sum_{i\in C} m_i q_{ij} = -x m_j, j \in C \qquad (1)$$

The notions of x invariant distribution and quasi stationary distribution are intimately related. Indeed, combining [14 & 15]**,** we can state the following.

Theorem (1):

Let $\mathcal{K}$be a birth death process with killing such that absorption at$-1$is certain. If $m \coloneqq (m_j, j \in C)$is a Quasi stationary distribution, then $m$is $x$invariantfor Q for some $x > 0$. Conversely, if $m$is $x$invariant for Q, then m is a Quasi stationary distribution if and only if

$$x = \sum_{j\in C} m_j \gamma_j \qquad (2)$$

We note that summing (1) over all $j \in C$results in (2) if the interchange of summation would be justified, which, however, is not the case in general.

[19] showed that if $(m_j, j \in C)$ is a Quasi stationary distribution, and hence$x$invariant for Q for some x, then x must be in the interval $0 < x \leq \alpha$, where α is the decay parameter of $\mathcal{K}$ in C. It follows that, besides certain absorption, $\alpha > 0$ is necessary for the existence of a quasi stationary distribution. In summary, if $\alpha > 0$ and absorption is certain, then, in order to find all quasi stationary distributions for $\mathcal{K}$, we have to find all proper distributions $(m_j, j \in C)$ which constitute a solution of (1) for some x, $0 < x \leq \alpha$, and satisfy (2).

2.2. Quasi Stationary Distributions:

Considering the recurrence relation of the polynomial sequence $\{R_n(x)\}$, the solution of the system of (1) is readily seen to be given by

$$m_j = m_0 \pi_j R_j(x), j \in C \qquad (3)$$

where m_0 is some constant. To obtain all Quasi stationary distributions, we thus have to find out for which values of x, $0 < x \leq \alpha$, the quantities m_j of (3) constitute a proper distribution with an appropriate choice of m_0, and satisfy (2). So the following three conditions have to be satisfied.

(i) We must have $m_j \geq 0$ for all j, and hence$R_j(x) \geq 0$ for all j. But this is a consequence of our assumption $x \leq \alpha$, which implies that $R_j(x) > 0$ for all j.

(ii) The sum $\sum_{j\in C} \pi_j R_j(x)$ must be finite, so that $(m_j, j \in C)$ becomes a proper distribution by choosing $m_0^{-1} = \sum_{j\in C} \pi_j R_j(x)$

(iii) Condition (2) must be satisfied, that is, if the previous requirements are met, we must have

$$x \sum_{j\in C} \pi_j R_j(x) = \sum_{j\in C} \gamma_j \pi_j R_j(x) \qquad (4)$$

Summarizing the preceding, we can state the following theorem.

Theorem (2):

Let $\mathcal{K}$ be a birth death process with killing such that absorption at -1 is certain. If $\alpha = 0$, there is no Quasi stationary distribution for $\mathcal{K}$. If $\alpha > 0$, then $(m_j, j \in C)$ is a Quasi stationary distribution for $\mathcal{K}$ if and only if there is a real number $x, 0 < x \leq \alpha$, such that

$$x\sum_{j\in C}\pi_j R_j(x)=\sum_{j\in C}\gamma_j\pi_j R_j(x)<\infty \quad (5)$$

and $m_j = m_j(x), j \in C$, where

$$m_j(x) \coloneqq m_0(x)\pi_j R_j(x), j \in C, m_0^{-1}=\sum_{j\in C}\pi_j R_j(x) \quad (6)$$

Lemma (1):

Let $0 < x \leq \alpha$. Then (5) is satisfied if and only if both

$$\sum_{j\in C}\pi_j R_j(x)<\infty \text{ or } \sum_{j\in C}\gamma_j\pi_j R_j(x)<\infty \quad (7)$$

and

$$\lim_{j\to\infty}\lambda_j\pi_j\left(R_{j+1}(x)-R_j(x)\right) \quad (8)$$

Unfortunately, it does not seem possible to give a general condition in terms of the rates of the process for (7) and (7) to be valid. However, more can be said by imposing some additional restrictions on the rates. In the following subsections, some special cases will be therefore discussed.

2.3. Special Case: Finitely Many Positive Killing Rates:

Let us first consider the situation in which $\gamma_j > 0$ for only finitely many states $i \in C$. In this case**,** (7) is trivially satisfied. Actually, both sums in (7) converge, as appears from the next lemma.

Lemma (2):

Let $\mathcal{K}$ be a birth death process with killing for which absorption at $-1$is certainand $\gamma_j > 0$for only finitely many states $i \in C$. Then $\sum_{j\in C}\pi_j R_j(x)<\infty$ for all x in the interval $0 < x \leq \alpha$.

Proof:

When $\gamma_j > 0$ for only finitely many states $i \in C$, Now we assume

$$\sum_{k=0}^{\infty}\frac{1}{\lambda_k\pi_k}=\infty \quad (9)$$

Now let $0 < x \leq \alpha$and suppose $\sum_{j\in C}\pi_j R_j(x)$ diverges. Since $\sum_{j\in C}\gamma_j\pi_j R_j(x)$ converges, we then have

$$\sum_{j=0}^{k}(\gamma_j - x)\pi_j R_j(x)\to-\infty \text{ as } k\to\infty$$

so that**,** by (9), $R_j(x)$ must be negative for j sufficiently large. But this is a contradiction, since $R_j(x) > 0$ for all j if $x \leq \alpha$. So the sum $\sum_{j\in C}\pi_j R_j(x)$ must be finite.

We conclude that $(m_j, j \in C)$, where $m_j(x)$ denotes the quantity defined in (6), constitutes a proper distribution for all $x$in the interval $0 < x \leq \alpha$. However, it is notnecessarily true that (8), and hence (5), is satisfied. In the special case of a pure birth death process ($\gamma_i = 0$ for all $i > 0$ and $\gamma_0 > 0$), a necessary and sufficient condition for (5) to be valid for all $x$in the interval $0 < x \leq \alpha$is that the sum

$$\sum_{n=0}^{\infty}\frac{1}{\lambda_n\pi_n}\sum_{j=n+1}^{\infty}\pi_j \quad (10)$$

should be divergent [17]. The proof of this result relies on the fact that the polynomials

$$\lambda_n\pi_n\left(R_{n+1}(x)-R_n(x)\right), n \geq 0$$

are themselves orthogonal with respect to a probability measure on $[0,\infty)$. This property is lost as soon as one leaves the setting of the pure birth death process, but if $\gamma_i > 0$ foronly finitely many states $i \in C$, we can get around this problem.

Theorem (3):

Let $\mathcal{K}$ be a birth death process with killing for which absorption at -1 is certain and $\gamma_i > 0$for only finitely many states $i \in C$. If $\alpha > 0$and the sum (10) diverges, then $(m_j(x), j \in C)$, with $m_j(x)$ given by (7), constitutes a Quasi stationary distribution for all $x$in the interval $0 < x \leq \alpha$.

We have relegated the proof of this theorem to the appendix, since it requires techniques which are not related to the central issues of this section. Let us now assume that the sum (10) is convergent. In a pure birth death process, we must then have $\alpha > 0$, and there is precisely one Quasi stationary distribution, namely, $(m_j(\alpha), j \in C)$. In the present, more general setting, we cannot exclude the possibilities that $\alpha = 0$ and, if $\alpha > 0$, that there are several values of $x$inthe interval $0 < x \leq \alpha$ such that $(m_j(x), j \in C)$ constitutes a Quasi stationary distribution, but in any case, we can show the following.

Theorem (4):

Let $\mathcal{K}$ be a birth death process with killing for which absorption at -1 is certain and $\gamma_i > 0$ for only finitely many states $i \in C$. If $\alpha > 0$ and the sum (10) converges, then $(m_j(\alpha), j \in C)$, with $m_j(\alpha)$ given by (6), constitutes a Quasi stationary distribution.

Proof:

We know that (4) is satisfied for $x = \alpha$, although bothsums may be infinite. However, Lemma (2) tells us that under the prevailing conditions, the sums must be finite. The result follows by Theorem (2).

2.4. Special Case: Bounded Birth and Death Rates:

We will next consider the setting in which

$$\lambda_i + \mu_i \leq M < \infty, i \in C \quad (11)$$

for some $M \in R^+$. As usual, $m_j(x)$ denotes the quantity defined in (6) and we will tacitly assume that $0 < x \leq \alpha$.

Since $\lambda_j + \pi_j = \mu_{j+1}\pi_{j+1}$, we have

$$\lambda_j \pi_j \left(R_{j+1}(x) - R_j(x)\right) = \mu_{j+1}\pi_{j+1}R_{j+1}(x) - \lambda_j \pi_j R_j(x)$$

which tends to zero if λ_j and μ_j are bounded and $\sum_{j\in C} \pi_j R_j(x)$ converges. So, by Lemma (1), the condition (5) for $\left(m_j(x), j \in C\right)$ to be a Quasi stationary distribution is fulfilled if $\sum_{j\in C} \pi_j R_j(x) < \infty$. But we can do somewhat better as follows.

Theorem (5):

Let $\mathcal{K}$be a birth death process with killing satisfying (11), for which absorption at -1 is certain. If $0 < x \leq \alpha$ and $\sum_{j\in C} \pi_j R_j(x) < \infty$ then $\left(m_j(y), j \in C\right)$ is a Quasi stationary distribution for all $y$in the interval $x \leq y \leq \alpha$.

Proof:

In [7], We have $0 < \sum_{j\in C} \pi_j R_j(y) \leq \sum_{j\in C} \pi_j R_j(x)$ if $x \leq y \leq \alpha$, so $\left(m_j(y), j \in C\right)$ is a Quasi stationary distribution for all y in the interval $x \leq y \leq \alpha$ if $\sum_{j\in C} \pi_j R_j(x) < \infty$.

We conclude that if absorption is certain, $\alpha > 0$, and the birth and death rates are bounded, then either there is no Quasi stationary distribution or $\left(m_j(x), j \in C\right)$ constitutes a Quasi stationary distribution for all x in an interval of the type $0 < a \leq x \leq \alpha$ (allowing for $a = \alpha$), or of the type $0 \leq a < x \leq \alpha$. If there are infinitely many Quasi stationary distributions, that is, $a < \alpha$, then $\left(m_j(x), j \in C\right)$ need not be a Quasi stationary distribution for all $x$in the interval $0 < x \leq \alpha$, so a can be strictly positive.

An example of this type of behavior is given in this section. We will first construct a process such that a Quasi stationary distribution which is $x$invariant exists if and only if $a < x \leq \alpha$ for some $a > 0$. Indeed, let $\mathcal{K}$ be a birth death process with killing with birth, death, and killing rates λ_i, μ_{i+1} and $\gamma_i, i \in C$ respectively, q-matrix Q and decay parameter α. Next, choose $\gamma > 0$ and let $\tilde{\mathcal{K}}$ be the birth death process with killing with transition rates $\tilde{\lambda}_i := \lambda_i, \tilde{\mu}_{i+1} := \mu_{i+1}, i \in C$ and

$$\tilde{\gamma}_i := \gamma + \gamma_i, i \in C$$

and matrix $\tilde{Q}$. One might interpret $\tilde{\mathcal{K}}$ as the superposition of $\mathcal{K}$ and an independent Poisson killing process of rate γ. Obviously, the transition probabilities of $\tilde{\mathcal{K}}$ and $\mathcal{K}$ arerelated as

$$\tilde{P}_{ij}(t) = e^{-\gamma t} P_{ij}(t), i, j \in C, t \geq 0$$

whence the decay parameter$\tilde{\alpha}$ of $\tilde{\mathcal{K}}$ satisfies $\tilde{\alpha} = \gamma + \alpha$. It is evident from (1) and Theorem (1) that an x invariant Quasi stationary distribution for $\mathcal{K}$ is a $(\gamma + x)$ invariant Quasi stationary distribution for $\tilde{\mathcal{K}}$, and vice versa. Now, if we choose $\mathcal{K}$ such that for each $x$in the interval $0 < x \leq \alpha$ there exists a Quasi stationary distribution then for each$\tilde{x}$in the interval $\gamma < \tilde{x} < \tilde{\alpha}$ there exists an$\tilde{x}$invariant Quasi stationary distribution for $\tilde{\mathcal{K}}$, but there are no$\tilde{x}$invariant Quasi stationary distributions for $\tilde{\mathcal{K}}$ with $\tilde{x} \leq \gamma$, since an x invariant quasi stationary distribution for $\mathcal{K}$ must have $x > 0$. Thus $\tilde{\mathcal{K}}$ has the required property, with $a = \gamma$.

Our final example is the process $\mathcal{K}$ with birth, death, and killing rate

$$\lambda_i = \lambda, \mu_i = \mu I_{\{i>0\}}, \quad \lambda_i = \lambda, \mu_i = \mu I_{\{i>0\}}, \quad i \in C$$

for some constants $\lambda > 0, \mu > 0$ and $\gamma > 0$, where I_E denotes the indicator function of anevent E. So killing may occur from any state except state 0. W**e** find that the decay parameter for this process is given by

$$\alpha = \begin{cases} \frac{\lambda\gamma}{\mu+\gamma} & , \mu + \gamma \geq \sqrt{\lambda\mu} \\ \gamma + \left(\sqrt{\lambda} - \sqrt{\mu}\right)^2 & , \mu + \gamma < \sqrt{\lambda\mu} \end{cases}$$

while

$$\pi_j R_j(x) = (-1)^j \left(\frac{\lambda}{\mu}\right)^{j/2} \left(U_j(y) + \eta U_{j-1}(y)\right), j \geq 0$$

where

$$y := \frac{x - \lambda - \mu - \gamma}{2\sqrt{\lambda\gamma}}, \quad \eta := \frac{\mu + \gamma}{\sqrt{\lambda\gamma}}$$

and $U_j(\cdot)$ denotes the j[th] Chebyshev polynomial of the second kind, that is,

$$U_j(y) = \frac{z^{j+1} - z^{-(j+1)}}{z - z^{-1}}, j \geq 0$$

with z such that $y = (1/2)\left(z + z^{-1}\right)$. Evidently, absorption is certain. Moreover, since λ_i and μ_i are bounded, we can employ Theorem (5) and conclude that we must determine all $x$such that $0 < x \leq \alpha$ and $\sum_{j\in C} \pi_j R_j(x) < \infty$ in order to find all Quasi stationary distributions. So let $0 < x \leq \alpha$. Considering that

$$\frac{\lambda\gamma}{\mu+\gamma} = \lambda + \mu + \gamma - \sqrt{\lambda\mu}\left(\eta + \eta^{-1}\right) \leq \gamma + \left(\sqrt{\lambda} - \sqrt{\mu}\right)^2$$

we have $0 < x \leq \lambda + \mu + \gamma 2\sqrt{\lambda\mu}$, and hence $= (1/2)\left(z + z^{-1}\right) \leq -1$. It is therefore no restriction of generality to assume $z \leq -1$. Moreover, we can write

$$\pi_j R_j(x) = \begin{cases} \left(\frac{\lambda}{\mu}\right)^{j/2} \left\{\frac{(-z)^j(z+\eta)-(-z)^{-j}(\eta+z^{-1})}{z-z^{-1}}\right\}, z < -1 \\ \left(\frac{\lambda}{\mu}\right)^{j/2}\{1+(1-\eta)j\} \qquad , z = -1 \end{cases}$$

so that $\sum_{j\in C}\pi_j R_j(x)$ diverges unless either $z \neq -\eta$ and $-z\sqrt{\lambda/\mu} < 1$, or $z = -\eta$ and $\eta^{-1}\sqrt{\lambda/\mu} = \lambda/(\mu+\gamma) < 1$. We now discern the following three cases.

(i) If $\lambda \geq \mu + \gamma$, then $-z\sqrt{\lambda/\mu} > 1$ and $\eta^{-1}\sqrt{\lambda/\mu} \geq 1$. Hence $\sum_{j\in C}\pi_j R_j(x)$ diverges.

(ii) If $\lambda < \mu + \gamma$ (and hence $\eta > 1$) and $x < \alpha$, then $y < -(1/2)(\eta + \eta^{-1})$ and hence $z < -\eta$, so that $z \neq -\eta$ and $-z\sqrt{\lambda/\mu} < 1$. Again it follows that $\sum_{j\in C}\pi_j R_j(x)$ diverges.

(iii) If $\lambda < \mu + \gamma, x = \alpha$ then $z = -\eta$ and $\eta^{-1}\sqrt{\lambda/\mu} < 1$. So now we have $\sum_{j\in C}\pi_j R_j(x) < \infty$.

Concluding, there is no Quasi stationary distribution if $\lambda \geq \mu + \gamma$, and there is precisely one Quasi stationary distribution $(m_j, j \in C)$ where

$$m_j = m_j(\alpha) = \left(1 - \frac{\lambda}{\mu+\gamma}\right)\left(\frac{\lambda}{\mu+\gamma}\right)^j, j \geq 0 \qquad (12)$$

if $\lambda < \mu + \gamma$, that is $\alpha < \gamma$.

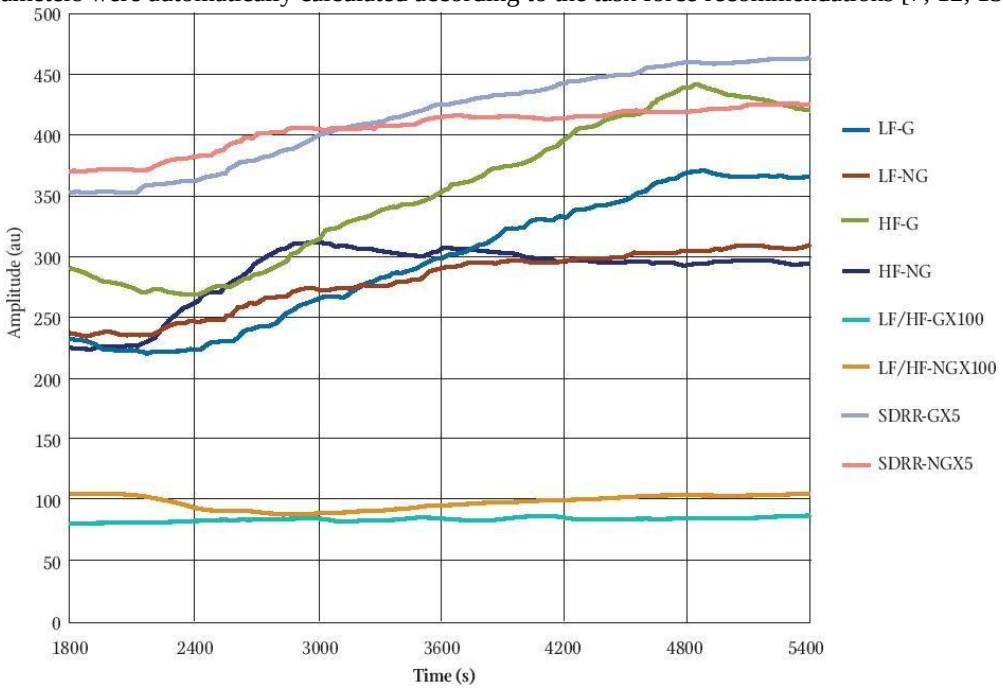
3. Example:

The health status of subjects was determined using the Health History Inventory (HHI) [3]. The results presented in this paper are those of 28 relatively healthy subjects (48.11 ± 14.48; average age ± standard deviation [SD]). These subjects were equally divided among men and women: 14 men (45.43 ± 13.62, range 25-66), and 14 women (50.79 ± 15.32, range 26-78). Informed consent was obtained from all subjects prior to their participation. The Biomedical Research Institute of America provided Institutional Review Board supervision of the project. Exclusion criteria were: 1) pregnancy; 2) age under 18 or over 80; 3) taking pain, anti-inflammatory, sedative, or prescription sleeping medications (less than 5 days prior to testing); 4) taking psychotropic drugs or being diagnosed with mental disorder; 5) recent surgery (less than 1 year); 6) documented life-threatening disease (such as cancer, AIDS [acquired immune deficiency syndrome], etc); 7) consumption of alcohol within 48 hours of participation; and 8) use of recreational drugs. Past pilot projects suggested that the resulting relatively healthy subjects may be more responsive to short term grounding.

The HRV parameters calculated from electrocardiogram recordings during the study were as follows:

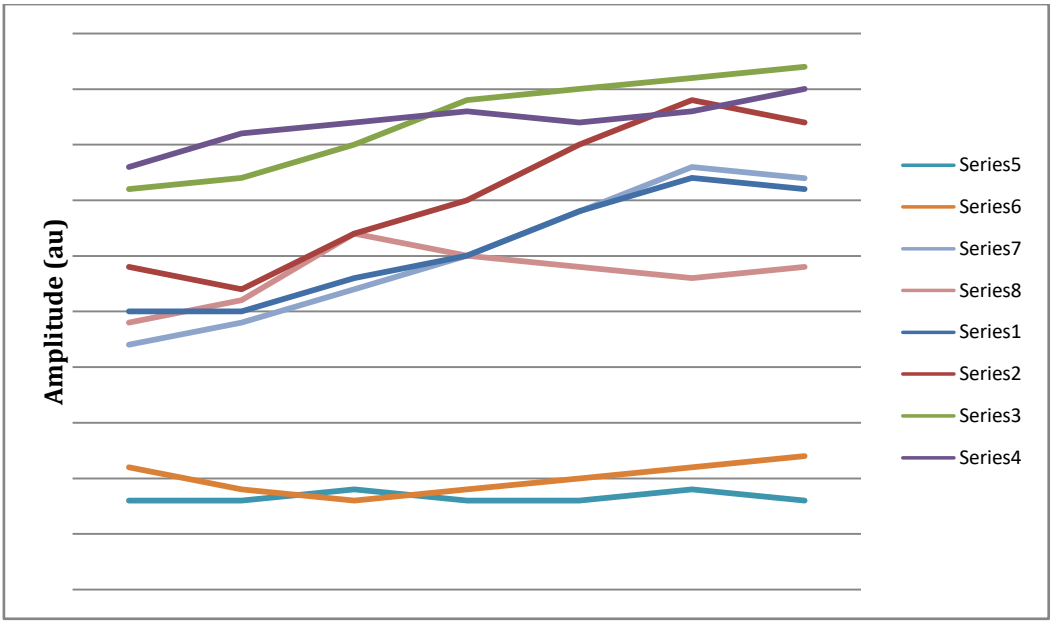
- The standard deviation of R-R intervals (SDRR, standard deviation of R peak to R peak intervals, also known as standard deviation of normal to normal intervals or SDNN)
- 3 spectral components of the power spectrum density (PSD, the square of the Fast Fourier Transform of the R-R intervals): low frequencies (LF), high frequencies (HF), and very low frequencies (VLF)
- The ratio LF/HF

These parameters were automatically calculated according to the task force recommendations [7, 12, 13 & 16].



LF = Low Frequencies, HF = High Frequencies, SDRR = Standard Deviation of R-to-R Peak Intervals, G = Grounded, NG = Not Grounded, X100 = Values Multiplied by 100, X5 = Values Multiplied by 5, (s) = Seconds

Figure (1): Grounded and Non Grounded Group Results



Series 8 = Low Frequencies, Series 1 = High Frequencies, Series 3 = Standard Deviation of R-to-R Peak Intervals, Series 2 = Grounded, Series 7 = Not Grounded, Series 6 = Values Multiplied by 100, Series 4 = Values Multiplied by 5, Series 5 = Seconds

Figure (2): Grounded and Non Grounded Group Results using Gamma Distribution

4. Conclusion:

Finally we estimate the heart rate variability with grounded & non grounded groups using birth death process through gamma distribution. By using gamma distribution the mathematical model gives the result as same as the medical report. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; ($i.e$) the results coincide with the mathematical and medical report.

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