



HESITANCY FUZZY GRAPHS BASED TIME MINIMIZED EMERGENCY ROUTE (TiMER)

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Abstract:

Ambulance service is one of the essential life saving public transport systems to assist the accident victims in emergency through this paper, an algorithm is proposed based on fuzzy principles to find a Time Minimized Emergency Route (TiMER) to rush the accident victims to the preferred hospitals. This paper incorporates Hesitancy Fuzzy Graphs to depict the nature of the path which is characterized with the linguistic variables such as downtown, road-weather condition, carriageway spacing, and traffic congestion.

Key Words: Hesitancy Fuzzy Graphs (HFGs), Urgency Degree, Berge Fuzzy Graph & TiMER

1. Introduction:

Fuzzy set theory has many applications in the real life situations. Fields such as Engineering, Robotics, Artificial Intelligence, Economics, Psychology, Social Science and Medicine are the few where techniques involving fuzzy principles are extensively used. Many models have been derived in the above mentioned fields with the help of fuzzy sets. Decision making is the prime goal in all such models. This paper enables the people in ambulance service sector to make a quick yet life saving decision to reach the victims to the preferred hospitals within a minimum duration of time.

On road, factors such as downtown area, road-weather condition, carriageway spacing and traffic congestion are very hard to quantify. These factors are complex in nature to discuss and enumeration is not so easy because of its unreliability. Often other factors such as weaving (routes branching out), bus stops, institutions; distance, road bumps (family of traffic calming devices) and road humps (rounded traffic calming devices) are also highly influencing the reliability and suitability of road.

This paper particularly analyses the road ways and ambulance services in Velachery, South Chennai, Tamil Nadu, India. A linguistic questionnaire (Appendix I) was prepared and opinion gathered from 100 people belonging to different sections of the society in Velachery area. Their subjective descriptions are quantified with the help of Fuzzy Matlab toolbox. The concept of Hesitancy Fuzzy Graphs (HFGs) is employed to analyze the arguments given by the road users.

Hesitancy Fuzzy Graphs (HFGs) [12 - 17] was introduced by T. Pathinathan et.al in order to capture the common intricacy that occurs during a selection of membership degree of an element from some possible values that makes one to hesitate. This paper uses Hesitancy Fuzzy Graphs (HFGs) to deal with the common hesitant situation and helps one to choose a Time Minimized Emergency Route (TiMER) to transport accident victims. Index Matrix (IM) representation of Hesitancy Fuzzy Graph is used to figure out the hesitant degree along with the membership and non-membership degrees of each entry (incidence value between the vertices).

The paper is organized as follows: Section Two describes the concept of Hesitancy Fuzzy Graphs (HFGs) and some of the associated basic definitions of hesitancy fuzzy graphs are defined. Section Three gives the insight of edge degree of a hesitancy fuzzy graphs and the urgency (density) degree of a hesitancy fuzzy graph. Section Four introduces the application part of Time Minimized Emergency Route

(TiMER) graph model which uses hesitancy fuzzy graphs extensively and also it discusses the MATLAB FUZZY TOOLBOX to synthesize the subjective opinions on choosing an emergency route. Section Five provides an illustrative example of the Time Minimized Emergency Route (TiMER) graph model with the case study and which is followed by conclusion in section Six.

2. Preliminaries:

This section contains some basic definitions and examples on Hesitancy Fuzzy Graphs and the other related concepts.

Definition 2.1 (Fuzzy Graph) [18]

Let V be a non empty set. A fuzzy graph is a pair of functions $G(\sigma, \mu)$ where σ is a fuzzy subset of V , μ is a symmetric fuzzy relation on σ .

$$\begin{aligned} \sigma &: V \rightarrow [0,1] \\ \mu &: V \times V \rightarrow [0,1] \\ \text{such that } \mu(u,v) &\leq \sigma(u) \wedge \sigma(v) \forall u, v \in V. \end{aligned}$$

The underlying crisp graph of the fuzzy graph $G(\sigma, \mu)$ is denoted as $G^* : (\sigma^*, \mu^*)$ where σ^* is referred to as the nonempty set V of nodes and $\mu^* = E \subseteq V \times V$. The crisp graph (V, E) is a special case of the fuzzy graph G with each vertex and edge of (V,E) having degree of membership 1.

Definition 2.2 (Berge Fuzzy Graph) [6]

A graph in the sense of Berge is one such that $E_1 = E_2 = E$ countable and is formed by the subset of ordered pairs $(x, y) \in G \subset E \times E$, such that

- (i) $G \cap \bar{G} = \phi$
- (ii) $G \cup \bar{G} = E \times E$

Also, this can be extended to a fuzzy concept by changing the two valued ordered pair relation into a multi-valued ordering. For instance, the example below shows the multi-valued weights associated with each edge between their respective incident vertices.

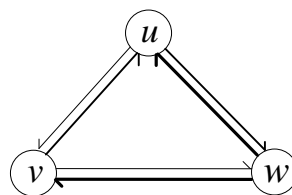


Figure 1: Berge Fuzzy Graph

In the above directed graph, the edges between the vertices shows multi-valued weights.

Definition 2.3 Hesitancy Fuzzy Graph [13]

A Hesitancy Fuzzy Graph is of the form $G = (V, E)$, where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0,1], \gamma_1 : V \rightarrow [0,1]$ and $\beta_1 : V \rightarrow [0,1]$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ respectively and

$$\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1 \quad \forall v_i \in V,$$

where $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$ and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ -----(1)

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0,1], \gamma_2 : V \times V \rightarrow [0,1]$ and $\beta_2 : V \times V \rightarrow [0,1]$ are such that,

$$\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j)) \text{ -----(2)}$$

$$\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j)) \text{ -----(3)}$$

$$\beta_2(v_i, v_j) \leq \min(\beta_1(v_i), \beta_1(v_j)) \text{ -----(4)}$$

And $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1 \quad \forall (v_i, v_j) \in E$ -----(5)

Notations:

- I. $\langle v_i, \mu_{1i}, \gamma_{1i}, \beta_{1i} \rangle$ denotes the vertex, degree of membership, non-membership and hesitancy of the vertex v_i .
- II. $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij}, \beta_{2ij} \rangle$ denotes the edge, degree of membership, non-membership and hesitancy of the edge relation $e_{ij} = (v_i, v_j)$ on V .

3. Degree of an Edge:

Definition 3.1 (Degree of an Edge in a Hesitancy Fuzzy Graph) [17]

Let $G : (\mu_{1i}, \gamma_{1i}, \beta_{1i}), (\mu_{2ij}, \gamma_{2ij}, \beta_{2ij})$ be a hesitancy fuzzy graph. Then the membership degree of an edge in a hesitancy fuzzy graph is defined by,

$$d_{\mu_2}(uv) = d_{\mu_2}(u) + d_{\mu_2}(v) - 2e_{\mu_2}(uv)$$

The non-membership degree of an edge in a hesitancy fuzzy graph is defined by;

$$d_{\gamma_2}(uv) = d_{\gamma_2}(u) + d_{\gamma_2}(v) - 2e_{\gamma_2}(uv)$$

The hesitancy degree of an edge in a hesitancy fuzzy graph is defined by;

$$d_{\beta_2}(uv) = d_{\beta_2}(u) + d_{\beta_2}(v) - 2e_{\beta_2}(uv)$$

where $(\mu_{1i}, \gamma_{1i}, \beta_{1i})$ represents the membership, non-membership and hesitancy degree of a vertex, and $(\mu_{2ij}, \gamma_{2ij}, \beta_{2ij})$ represents the membership, non-membership and hesitancy degree of an edge.

Definition 3.1 (Total Degree of an Edge in a Fuzzy Graph) [17]

Let $G : (V, E)$ be a fuzzy graph. The total degree of an edge $e = uv \in E$ is defined by $td_G(uv) = d_G(u) + d_G(v) - e(uv)$

Definition 3.2 (Total Degree of an Edge in a Hesitancy Fuzzy Graph) [16]

Let $G : (\mu_{1i}, \gamma_{1i}, \beta_{1i}), (\mu_{2ij}, \gamma_{2ij}, \beta_{2ij})$ be a hesitancy fuzzy graph. Then the membership value of total degree of an edge uv is given by, $d_{\mu_2}(uv) = d_{\mu_2}(u) + d_{\mu_2}(v) - e_{\mu_2}(uv)$.

The non-membership total degree of an edge in a hesitancy fuzzy graph is defined by;

$$d_{\gamma_2}(uv) = d_{\gamma_2}(u) + d_{\gamma_2}(v) - e_{\gamma_2}(uv)$$

The hesitancy total degree of an edge in a hesitancy fuzzy graph is defined by;

$$d_{\beta_2}(uv) = d_{\beta_2}(u) + d_{\beta_2}(v) - e_{\beta_2}(uv)$$

where, $(\mu_{1i}, \gamma_{1i}, \beta_{1i})$ represents the membership, non-membership and hesitancy degree of a vertex, and $(\mu_{2ij}, \gamma_{2ij}, \beta_{2ij})$ represents the membership, non-membership and hesitancy degree of an edge.

Definition 3.3 (Density of Hesitancy Fuzzy Graph) [15]

The density of an intuitionistic fuzzy graph $G = (V, E)$ is $D(G) = (D_{\mu}(G), D_{\gamma}(G), D_{\beta}(G))$, where,

$$D_{\mu}(G) \text{ is defined by } D_{\mu}(G) = 2 \left(\frac{\sum_{v_i, v_j \in V} \mu_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\mu_1(v_i) \wedge \mu_1(v_j))} \right), \text{ for } v_i, v_j \in V$$

$$D_{\gamma}(G) \text{ is defined by } D_{\gamma}(G) = 2 \left(\frac{\sum_{v_i, v_j \in V} \gamma_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\gamma_1(v_i) \vee \gamma_1(v_j))} \right), \text{ for } v_i, v_j \in V$$

$$D_{\beta}(G) \text{ is defined by } D_{\beta}(G) = 2 \left(\frac{\sum_{u, v \in V} \beta_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\beta_1(v_i) \wedge \beta_1(v_j))} \right), \text{ for } v_i, v_j \in V$$

Otherwise,

$$D(G) = (D_{\mu}(G), D_{\gamma}(G), D_{\beta}(G))$$

$$D(G) = \left(2 \left(\frac{\sum_{v_i, v_j \in V} \mu_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\mu_1(v_i) \wedge \mu_1(v_j))} \right), 2 \left(\frac{\sum_{v_i, v_j \in V} \gamma_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\gamma_1(v_i) \vee \gamma_1(v_j))} \right), 2 \left(\frac{\sum_{u, v \in V} \beta_2(v_i, v_j)}{\sum_{(v_i, v_j) \in E} (\beta_1(v_i) \wedge \beta_1(v_j))} \right) \right)$$

4. Application of Hesitancy Fuzzy Graph (HFG)

The role of Ambulance service is one of major importance in an emergency situation. This paper concerns on choosing the Time Minimized Emergency Route (TiMER) so that the victims are transported for the required medical care as soon as possible. The major factors that affect traffic and thereby influence the time duration taken by ambulance in reaching the hospital are distance, downtown area, humps and bumps, weaving, bus stops and existing schools and institutions on the road sides. The major task is to convert qualitative variables by defining suitable Membership Functions.

MATLAB FUZZY TOOLBOX (FMT) is basically used to establish mapping among the input variables. With the help of inference rules and fuzzy implication, the outcome is obtained. There are four main inference methods namely Mamdani method, Larsen method, Tsukamoto method and Takagi-Sugeno-Kang method (TSK method) [8]. We use Mamdani method in our work. In this method minimum operator is used as a fuzzy implication and max-min operator is used as the composition. These two operators are simple and most suited for our purpose.

4.1 Adaptation of the problem:

To choose the Time Minimized Emergency Route (TiMER) to reach the accident victim or victims to a preferred hospital; we have to analyze the factors that affect the

travel time in reaching the destination. The major factors and the sub factors affecting the time taken in reaching the destination listed below;

The major factors affecting the traffic are listed as follows:

1. Commercial/Market/Congested area (C/M/C area)
2. Weaving (routes branching out)
3. Carriageway (Width / single or double lane)
4. Distance
5. Congestion

The other factors, which are directly and indirectly lead to traffic jam or speed reduction, may be;

6. Road bumps (family of traffic calming devices)
7. Road humps (rounded traffic calming devices)
8. Road condition
9. Number of bus stops
10. Number of Institutions (School/College/Government offices)

The above mentioned first five factors (1-5) are the major factors that have strong influence on deciding the Time Minimized Emergency Route (TiMER). Sub factors (6-10) are the factors often considered to be an obstacle in reaching the destination. For the study, we have kept the major factors (1-5) as our input variables. Input variables are classified with the help of following linguistic descriptions (Table 1).

Table 1: Input and Output variables classification

Variables	Factors	Linguistic Description
Input Variables	C/M/C area	VL-Very low, L-Low, M-Moderate, H-High, VH-Very High
	Weaving	LF-Less Favorable, F-Favorable, VF-Very Favorable
	Carriageway	LS-Low Spacing, S-Spacing, HS-High Spacing
	Distance	LA-Low Agreeable, A-Agreeable, HA-High Agreeable
	Congestion	HJ-High Jammed, J-Jammed, LJ-Low Jammed
Output Variable	Satisfactory Level	HS-Highly Satisfactory, S-Satisfactory, LS-Low Satisfactory, NS-Not Satisfactory

4.3 Fuzzy Controller for Ambulance Service

Fuzzy controller for studying the nature of the road to analyze Time Minimized Emergency Route (TiMER) consists of three components. They are;

- (i) Fuzzy input variables
- (ii) Fuzzy inference Rules (or) Black Box (or) Hidden Box
- (iii) Fuzzy output variable

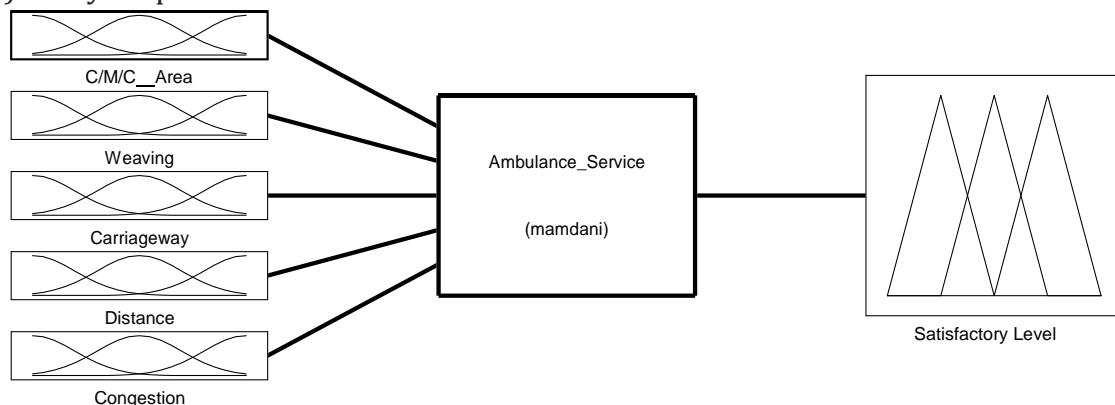


Figure 2: Fuzzy Controller for Ambulance Service

The linguistic variables (Table 1) which play a vital role in choosing a Time Minimized Emergency Route (TiMER) to reach the accident victim are considered to be the fuzzy input variables in the fuzzy controller.

When we consider the factor Commercial/Market/Congested area (C/M/C area), we have to consider sub factors like time, seasons (ordinary or festive) etc. and accordingly we use variables like Very High (VH), High (H), Moderately High (MH), Low (L) and Very Low (VL) in order to capture the whole reality of the factor C/M/C area. Due to the complex factors, it is very difficult to capture the reality. In order to make it more realistic, we use MATLAB to designate membership functions. We have adapted Triangular Fuzzy Membership Function (TFMF) [8] in order to capture the real picture of the factors involved.

Triangular Fuzzy Membership Function is the simplest form of the membership functions. It is used to specify a linguistic term providing the modal value of the considered term along with the lower and upper bounds [8]. In a similar way, we have to analyze the other factors such as weaving, carriageway spacing, distance and traffic congestion. We have used MATLAB FUZZY TOOLBOX to construct this ambulance fuzzy controller. The linguistic input variables and their membership functions are structured in the following way as in figure 2. We have adapted the triangular membership function to define the linguistic input variables and the MATLAB FIS Editor: Ambulance Service window (or) Mamdani FIS System (Figure 3 – 7) below shows the fuzzy membership functions for all the respective factors.

Our goal, the output variable, is to map a route in our study area with high level of satisfaction. Once again, the triangular fuzzy membership function is employed to define the nature of the output variable “satisfactory level” (Figure 8). The outputs we get depends on Fuzzy control Rules we have framed frame with the help of Expert’s opinion. The important task in constructing fuzzy controller involves programming the fuzzy inference rules. Fuzzy inference rules are the rules which govern the total system in decision making by analyzing each linguistic input variable with the output variable. Therefore, we mainly use two measuring scales namely nominal and ordinal scales. Nominal scale informs only about the categorical reality and ordinal scale tells us about order. However, the use of ordinal scale tells us about preference. Using preference relations Fuzzy inference rules are formed. The numbers of Fuzzy control rules depends on the number of fuzzy variables and fuzzy terms.

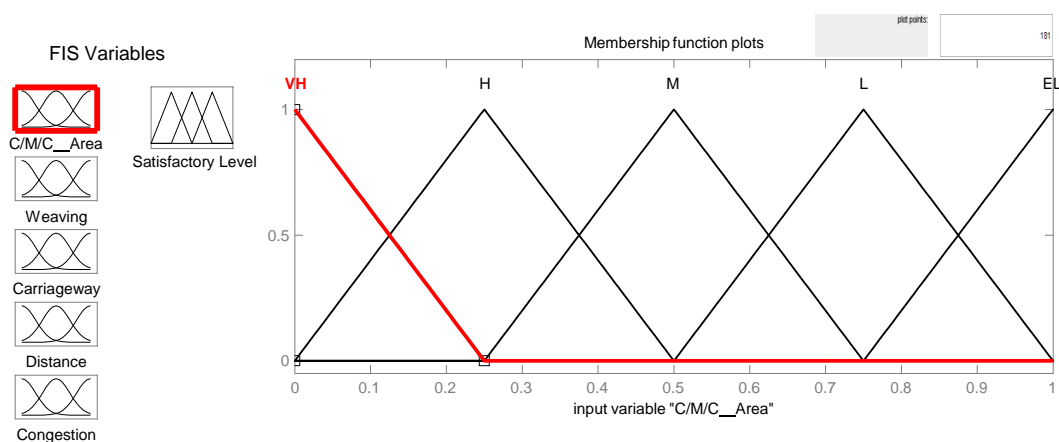


Figure 3: Fuzzy input membership function for C/M/C area

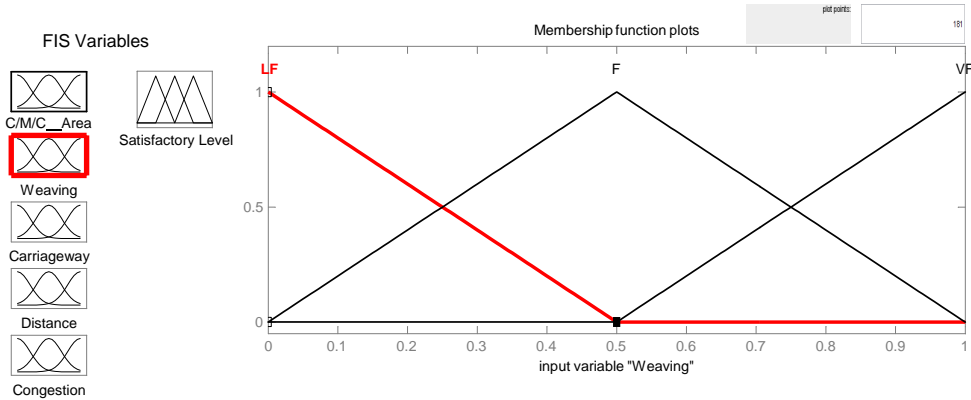


Figure 4: Fuzzy input membership function for Weaving

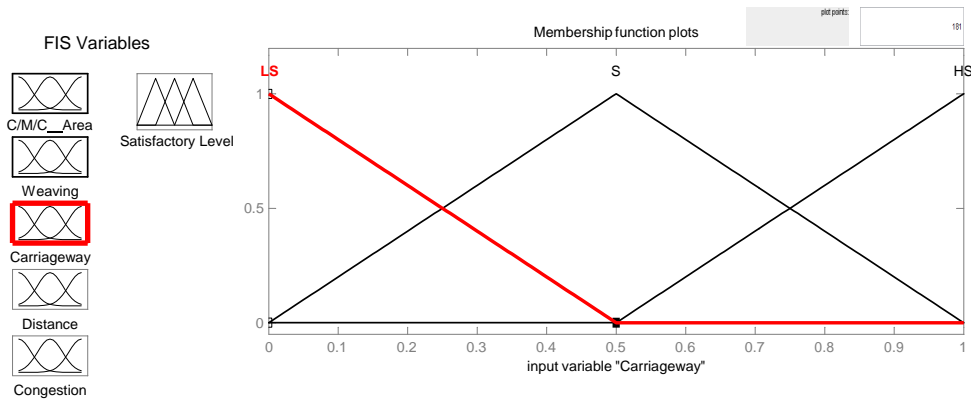


Figure 5: Fuzzy input membership function for Carriageway

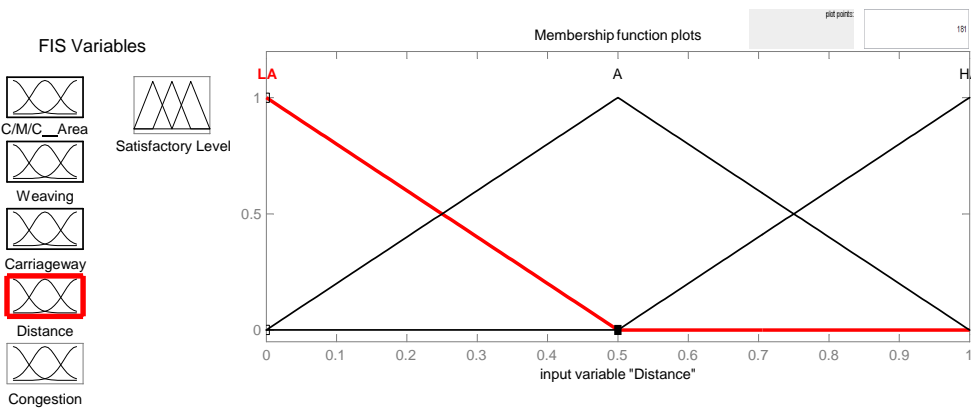


Figure 6: Fuzzy input membership function for Distance

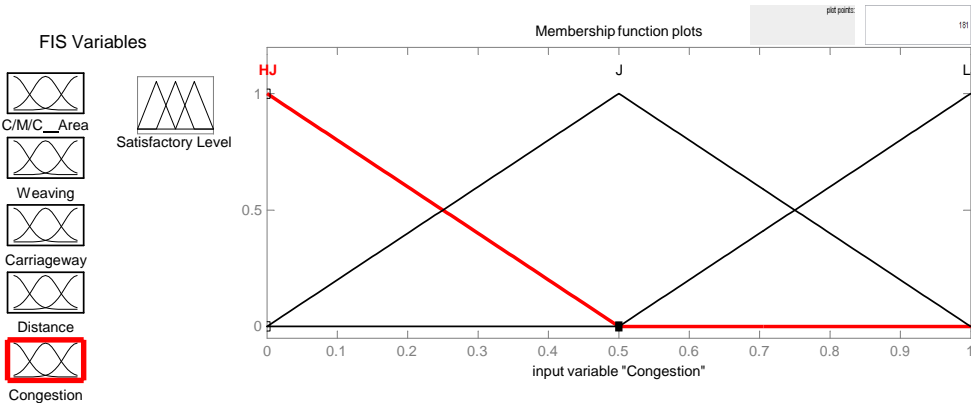


Figure 7: Fuzzy input membership function for Congestion

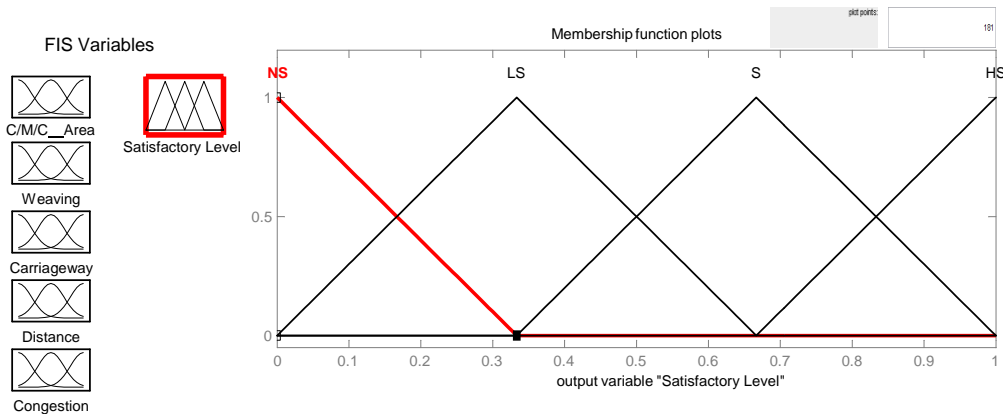


Figure 8: Fuzzy output membership function

The following are the few fuzzy inference rules which categorize various outputs based on the influence of fuzzy linguistic input variables. In order to capture the nuances, rules are framed with the help of all such possibilities of input variables to the output variable. The total of 405 rules are framed and programmed into the MATLAB FIS rule editor. The following are the few rules which are programmed into the FIS rule editor.

- ✓ If (C/M/C area is VH) and (Weaving is LF) and (Carriageway is LS) and (Distance is LA) and (Congestion is HJ) then (Output) is NS
- ✓ If (C/M/C area is M) and (Weaving is VF) and (Carriageway is LS) and (Distance is LA) and (Congestion is J) then (Output) is LS
- ✓ If (C/M/C area is L) and (Weaving is LF) and (Carriageway is HS) and (Distance is HA) and (Congestion is LJ) then (Output) is S
- ✓ If (C/M/C area is EL) and (Weaving is VF) and (Carriageway is HS) and (Distance is LJ) and (Congestion is HS) then (Output) is HS

On processing 405 rules by using Mamdani Fuzzy Inference System, the following rule viewer window (Figure 9) will open. By using this rule viewer window, we can generate outputs for various experts' linguistic inputs. Following this, 3D view of the input variables, which is obtained from MATLAB FUZZY TOOLBOX, is presented (Figure 10).

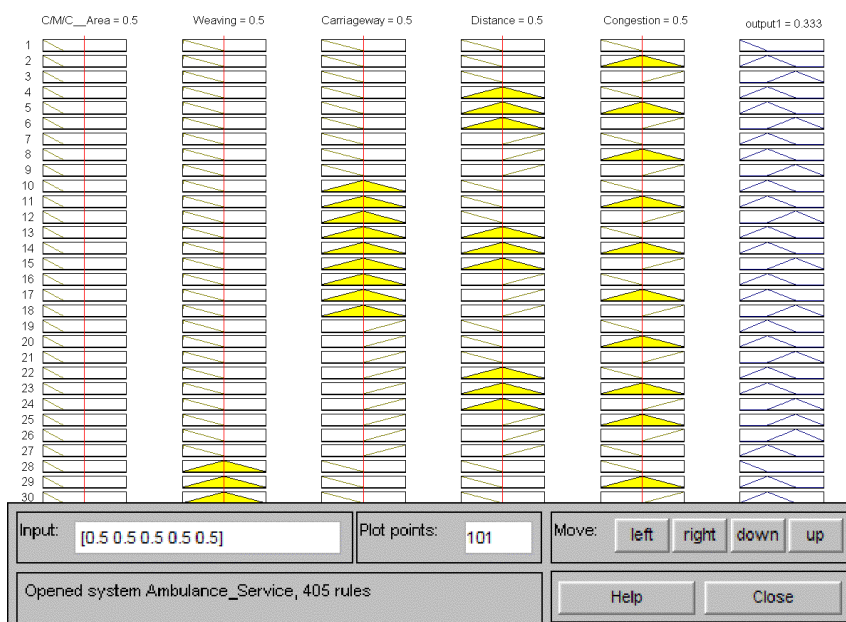


Figure 9: Fuzzy Rule Viewer Window

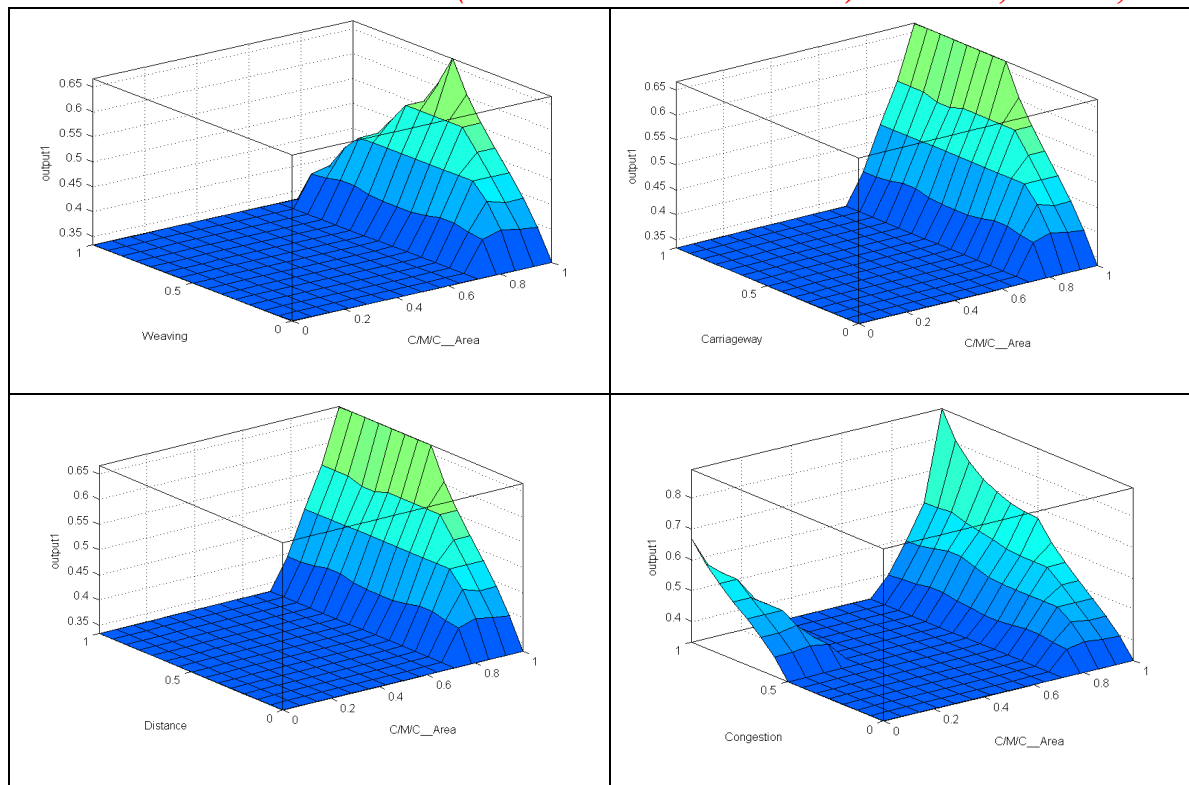


Figure 10: Fuzzy Surface Viewer

5. Case Study

Study area includes the region of Velachery of South Chennai. Velachery is one of the residential areas in South Chennai and registers rapid growth due to connectivity with OMR (a business class information technology corridor) and GST (National Highway 45 or also known as Grand Southern Trunk Road). In recent years, the number of people using roads has highly increased in and around Velachery, because of the presence of IT companies such as Tata Consultancy Services (TCS), Zoho Corp, AllSec Technologies and Sutherland Global Services. Also, Velachery has India's biggest shopping mall named Phoenix Market City. The goal is to reach the accident victim to the preferred hospital which offers modern and best suitable emergency treatment. In this paper, an example is derived for an arbitrary accident spot with the various paths by which the ambulance can take the victims to the hospital.

Gandhi Road Junction is considered to be the arbitrary accident spot and the ambulance driver has a plan to take the victim to the preferred super specialty hospital Xcellent Care Super Specialty Hospital (Private). Survey has been conducted from the frequent road users, ambulance service station; healthcare services in and around various parts of Velachery. Using the survey, we have obtained the suggestions given by the experts. Most of them suggested four different paths to reach out the hospital from the accident spot. The paths suggested by the experts are given below;

Table 2: Path with its routes

Path No	Route Description
P ₁	Velachery Road - Velachery Bypass Road
P ₂	Throwpathy Amman Koil Street
P ₃	Velachery Road, Dhandeeswaram - Dhandeeswaram 1 st Main Road
P ₄	Velachery Road, Dhandeeswaram - Velachery Road - Velachery Bypass Road

Based on the paths suggested by the experts the Hesitancy Fuzzy Graph (HFG) has been drawn along with the edge characterized by the notion of Berge Fuzzy Graph (BFG).

The Graph has been constructed with the vertex set $V = \{V_1, V_2, V_3, V_4, V_5\}$ represents the junction places

V_1 – Gandhi Road Junction

V_2 – Velachery Road and Velachery Bypass Road Junction

V_3 – Dhandeeswaram

V_4 – Vijaya Nagar Junction

V_5 – Xcellent Care Super Speciality Hospital (Private)

and the edge set $E = \{V_1V_2, V_1V_3, V_1V_5, V_2V_5, V_3V_4, V_3V_5, V_4V_5\}$ represents the path connectivity (Table) between the junctions. The vertex entries are calculated with the help of experts' subjective opinion about the junction characteristics like traffic flow and C/M/C area around the junction. Subjective opinions regarding the route characteristics are very hard to quantify and Hesitancy Fuzzy Graph is employed to measure the degree of membership in selecting a route, the degree of non-membership which shows the negativity in selection and the hesitancy degree which deals with the hesitant situation arise during the selection. Hence each vertex has the entries with the 3-tuple value denoting the membership, non-membership and hesitancy degree. The paths suggested by the experts are represented with the edge description in the following table (Table 3).

Table 3: Path with its edge description

Path No	Path	Edge Description
P_1	$v_1 - v_2$	Velachery Road
	$v_2 - v_5$	Velachery Bypass Road
P_2	$v_1 - v_5$	Throwpathy Amman Koil Street
P_3	$v_1 - v_3$	Velachery Road, Dhandeeswaram
	$v_3 - v_5$	Dhandeeswaram 1 st Main Road
P_4	$v_1 - v_3$	Velachery Road, Dhandeeswaram
	$v_3 - v_4$	Velachery Road
	$v_4 - v_5$	Velachery Bypass Road

The various characteristics related with the paths are studied extensively with the support of the observation made during the interview and the opinions given by the experts' are tabulated as follows (Table 4):

Table 4: Characteristics of the path on the study area

Path No	Path	Distance (in mts)	Road Capacity	Speed Bumps/Humps	Downtown Area	Traffic Congestion	All-weather road	No of Bus Stops	No of Educational Institutions	Weaving
P_1	$v_1 - v_2$	650	DSCR/USCR	-	M	TD	Moderate Defect	2	3	SW
	$v_2 - v_5$	700	DDCR	-	EH	TD	Low Defect	2	0	MW
P_2	$v_1 - v_5$	850	USCR	5	VL	TD	Low Defect	1	0	MW
P_3	$v_1 - v_3$	350	USCR	-	H	TD	High Defect	2	0	MW
	$v_3 - v_5$	1000	USCR	4	L	TD	Low Defect	1	0	MW
P_4	$v_1 - v_3$	350	USCR	-	H	TD	High Defect	2	0	MW
	$v_3 - v_4$	1400	USCR	-	H	TD	Moderate Defect	2	0	OSW
	$v_4 - v_5$	2100	DDCR	-	E	TD	Low Defect	2	0	MW

where, USCR – Undivided Single Carriageway Road, DSCR – Divided Single Carriageway Road, UDCR – Undivided Dual Carriageway Road, DDCR – Divided Dual Carriageway Road

EH – Extremely High, VH – Very High, H – High, PH –Possibly High, M - Moderate, PL – Possibly Low, L – Low, VL – Very Low, EL – Extremely Low

HD – High Defect, MD – Moderate Defect, LD – Low Defect congestion

TD – Time Dependent

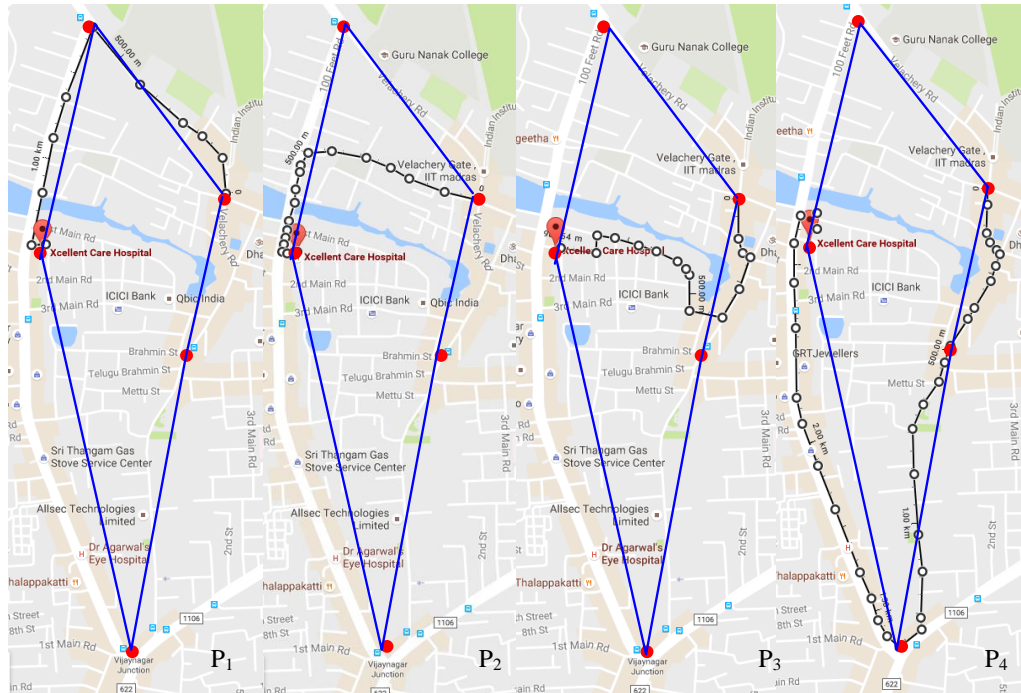


Figure 11: Various paths between source and destination node

The subjective opinions are processed with the help of MATLAB FUZZY TOOLBOX and the values of the membership, non-membership and hesitancy degree for each junction is calculated as follows;

Table 5: Hesitancy Fuzzy Input Values

Vertex	I ₁	I ₂	I ₃	I ₄	I ₅	Membership Degree	Non-membership Degree	Hesitancy Degree
v ₁	0.3	0.1	0.3	0.6	0.5	0.333	0.333	0.334
v ₂	0.7	0.8	0.8	0.8	0.8	0.527	0.272	0.201
v ₃	0.2	0.3	0.3	0.6	0.3	0.306	0.435	0.259
v ₄	0.1	0.2	0.3	0.7	0.4	0.325	0.482	0.193
v ₅	0.1	0.1	0.1	0.2	0.05	0.272	0.567	0.161

Based on the membership, non-membership and hesitancy degree calculated from the opinions, we have the following directed graph (Figure 12) with the vertices represented the junctions and the edges represented the strength of the path. The concept of Hesitancy Fuzzy Graph (Definition 2.3) is applied to find the strength (weight) of the path with its respective junctions.

Table 6: Membership value of the paths

Vertex	I ₁	I ₂	I ₃	I ₄	I ₅	Membership Degree
v ₁ - v ₂	0.2	0.05	0.1	0.3	0.2	0.306
v ₂ - v ₅	0.1	0.1	0.1	0.1	0.05	0.231
v ₁ - v ₅	0.05	0.05	0.1	0.1	0.05	0.215
v ₁ - v ₃	0.1	0.05	0.1	0.3	0.1	0.306
v ₃ - v ₅	0.05	0.05	0.1	0.1	0.05	0.215

$v_1 - v_3$	0.1	0.05	0.1	0.3	0.1	0.306
$v_3 - v_4$	0.05	0.1	0.1	0.3	0.2	0.306
$v_4 - v_5$	0.05	0.1	0.1	0.1	0.05	0.215

By the definition of the Hesitancy Fuzzy Graph developed by T. Pathinathan et.al, the degree of non-membership value is determined by taking the maximum level of disagreement towards the particular route. Also, the degree of hesitancy is determined by analyzing the level of state of confusion in choosing an emergency preferred route. The below tables (Table 7 and Table 8) show the respective degrees of non-membership and hesitancy degree over the selection of time minimized emergency route.

Table 7: Non-membership value of the paths

Vertex	I ₁	I ₂	I ₃	I ₄	I ₅	Non-Membership Degree
$v_1 - v_2$	0.5	0.6	0.5	0.4	0.3	0.333
$v_2 - v_5$	0.5	0.6	0.6	0.5	0.6	0.414
$v_1 - v_5$	0.2	0.1	0.2	0.3	0.2	0.306
$v_1 - v_3$	0.2	0.2	0.2	0.3	0.3	0.306
$v_3 - v_5$	0.2	0.2	0.2	0.3	0.1	0.306
$v_1 - v_3$	0.2	0.2	0.2	0.3	0.3	0.306
$v_3 - v_4$	0.1	0.2	0.3	0.6	0.3	0.306
$v_4 - v_5$	0.1	0.1	0.1	0.4	0.2	0.325

Table 8: Hesitancy degree of the paths

Vertex	I ₁	I ₂	I ₃	I ₄	I ₅	Hesitancy Degree
$v_1 - v_2$	0.05	0.05	0.05	0.05	0.05	0.175
$v_2 - v_5$	0.01	0.01	0.01	0.01	0.01	0.123
$v_1 - v_5$	0.05	0.01	0.05	0.0125	0.05	0.123
$v_1 - v_3$	0.05	0.0125	0.05	0.1	0.1	0.215
$v_3 - v_5$	0.01	0.01	0.01	0.01	0.01	0.123
$v_1 - v_3$	0.05	0.0125	0.05	0.1	0.1	0.215
$v_3 - v_4$	0.05	0.01	0.05	0.0125	0.05	0.175
$v_4 - v_5$	0.01	0.01	0.01	0.01	0.01	0.123

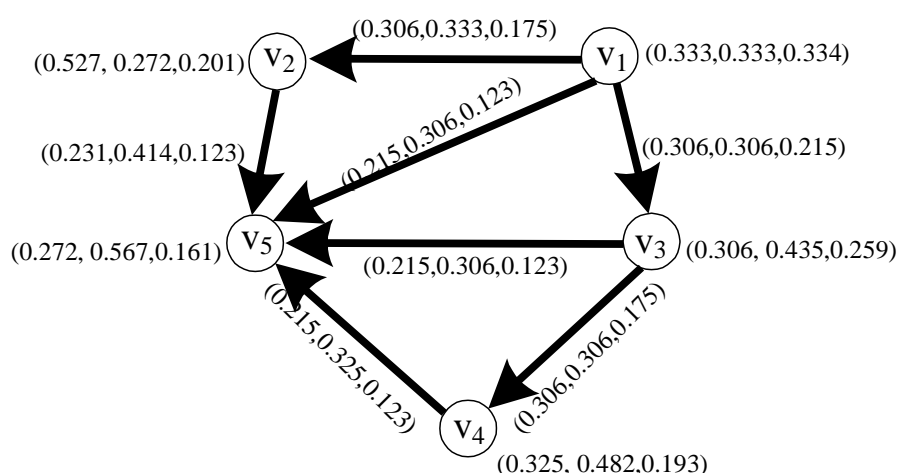


Figure 12: Hesitancy Fuzzy Graph

The above figure represents the various junctions connected to the preferred hospital along with its directed edges.

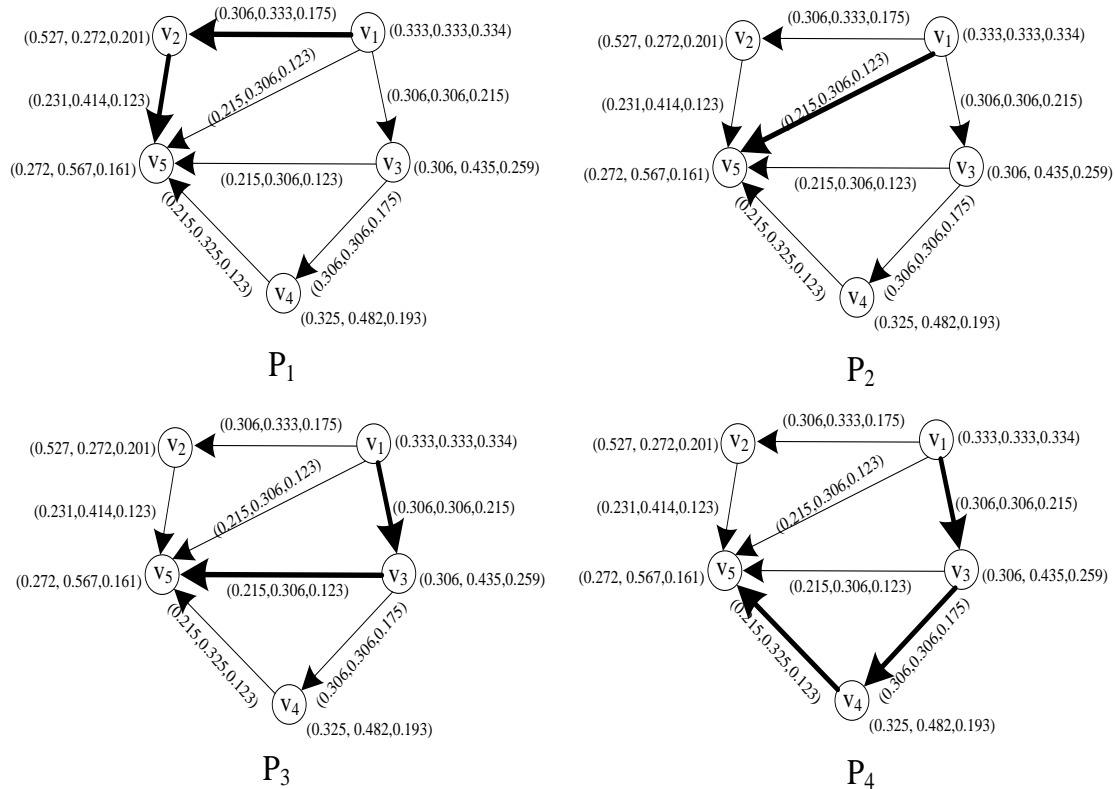


Figure 13: Graphs showing various paths

The above figure represents the various paths (P_1 , P_2 , P_3 and P_4) between the accident spot (v_1) and the preferred super specialty hospital (v_5). From the table and the figure, it is observed that the weight of each edge is found by the concept of hesitancy fuzzy graph introduced by T. Pathinathan et al [13]. The edge weights are tabulated (Table 9) as follows;

Table 9: Paths with its edge weights

Path No	Path	Edge weights
P_1	$v_1 - v_2$	$\langle 0.306, 0.333, 0.175 \rangle$
	$v_2 - v_5$	$\langle 0.231, 0.414, 0.123 \rangle$
P_2	$v_1 - v_5$	$\langle 0.215, 0.306, 0.123 \rangle$
P_3	$v_1 - v_3$	$\langle 0.306, 0.306, 0.215 \rangle$
	$v_3 - v_5$	$\langle 0.215, 0.306, 0.123 \rangle$
P_4	$v_1 - v_3$	$\langle 0.306, 0.306, 0.215 \rangle$
	$v_3 - v_4$	$\langle 0.306, 0.306, 0.175 \rangle$
	$v_4 - v_5$	$\langle 0.215, 0.325, 0.123 \rangle$

The urgency of each path to reach the destination depends on the degree of the vertices (junctions) connecting them along with its incident edges (route connecting the junction). The urgency degree is calculated by the definition (Definition 3.3). The below table (Table 10) shows the urgency degrees of each path associated with its membership, non-membership and hesitancy degree.

Table 10: Paths with urgency degrees

Path No	Path	Urgency degree
P ₁	v ₁ → v ₂ → v ₅	⟨1.7752, 1.66, 1.6464⟩
P ₂	v ₁ - v ₅	⟨1.5808, 1.079, 1.5279⟩
P ₃	v ₁ → v ₃ → v ₅	⟨1.8027, 1.2215, 1.6095⟩
P ₄	v ₁ → v ₃ → v ₄ → v ₅	⟨1.8710, 1.2628, 1.6737⟩

The above table shows the degree of urgency between the source (accident spot) and destination node (hospital). Also the degree of membership, non-membership and hesitancy degree of each path is defined and tabulated in the above table. From the above table, the result is observed as follows;

$$P_2 \succ P_1 \succ P_3 \succ P_4$$

i.e., the path (P₂) from the junction (v₁ - Gandhi Road) to the destination (v₅ - Xcellent Care Hospital) has the smallest amount of urgency (density) degree (means, reduces the stress by undertaking all kinds of obstacles in reaching the destination in an emergency) when compared to the other routes.

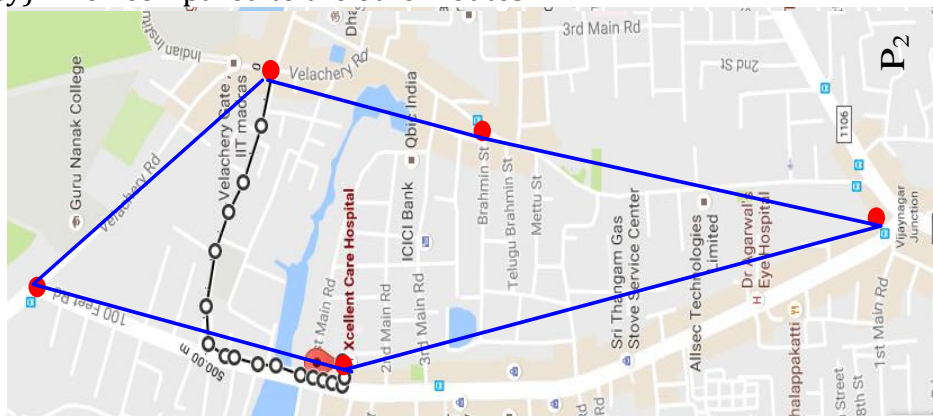


Figure 12: Path P₂ from V₁ to V₅ with least urgency degree

The membership value of the urgency degree gives the level of satisfaction over the path and also it shows the belief that the ambulance driver has towards that particular route. The non-membership value of the urgency degree predicts the level of dissatisfaction towards the particular route. Then importantly, hesitancy value of the urgency degree is the one which actually determines the agreement over the selection of route to reaching the destination.

Also, the path connects (v₁ - Gandhi Road) with the junction (v₂ - Guru Nanak College Junction) to the destination (v₅ - Xcellent Care Hospital) has the second smallest amount of urgency degree in reaching accident survivors to the need. The paths P₃ and P₄ have the highest degree of hesitancy over choosing the route to reach the destination. This is a case with five junctions representing vertices along its incident edges to discuss the preferred path by taking on-time decisions. This can also be extended to the large-sized network which involves computation.

6. Conclusion:

This paper particularly analyses the decision taken by the ambulance drivers, neighbors in an emergency situation to save the accident victims. Time Minimized Emergency Route (TiMER) model is introduced to study the problem of medical emergency in order to minimize the time in reaching the destination. The concept of Hesitancy Fuzzy Graph (HFG) is applied for choosing a Time Minimized Emergency Route (TiMER) to transport the accident victims to the preferred hospital. Also this

paper covers a case study to discuss the Time Minimized Emergency Route (TiMER) model in a small area of Velachery, South Chennai.

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Appendix - I

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PG & Research Department of Mathematics

Suggestions for Emergency Needs and Fast Approachable Routes by Ambulance Services in and Around Velachery, Chennai

Questionnaire:

General Information:

1. Name:
2. Age:
3. Gender
4. Marital Status:
5. Address:
6. Occupation:
7. Educational Qualification:
8. Economic status(Annual Income): (i) Less than Rs. 50,000; (ii) Rs. 50,000 – Rs. 1,00,000; (iii) Rs. 1,00,000 - GRs. 5,00,000 ;(iv) More than Rs. 5,00,000 .
9. Have you ever come across or heard any accident spots?
10. How do you wish to help the victims when you come across or you have heard about: like
 - (i) Informing police
 - (ii) Informing Ambulance;
 - (iii) Calling neighbours for help
 - (iv) Nobody
11. What is the reason for not attending?
 - (i) Informing police may cause unnecessary problem
 - (ii) Allergy to sight of the blood and blood covered area
 - (iii) Psychological fear to look at the accident spot
 - (iv) Having urgent or personal work
 - (v) Not interested
 - (vi) Has no money to spend.
12. (i) If your preference is informing ambulance then why other measures are less important?

Or

(ii) If your preference is informing police then why other measures are less important?

Or

(iii) If your preference is calling neighbours for help then why other measures are less important?

13. Will you make any follow up after admitting in to the hospital?

For Ambulance Services People:

1. Once you get the information what will be your first response?
2. Will you go on your own or wait for instruction from any authority?
3. Or will you call any other ambulance service?
4. Will you inform the hospital personnel for alert?
5. Where will you normally take the victims?

6. Which route will you prefer to reach the accident spot?
7. Do you have any procedure to handle the victims?
8. After looking at the condition of the accident victim will you change the hospital to reach?
9. What type of hospitals do you prefer to admit?
10. On which route do you take the patient to the hospital?

For Neighbours in and Around Velachery:

1. Have you ever seen any accident your area?
2. Had you rushed to the accident spot?
3. What type of help have you made?
4. What will be your first step to safeguard the life of the patient?
Call i) Ambulance (ii) Police (iii) Any other
5. Will you inform any of the victim's relative? How?
6. What type of service do you recommend to the victim?
7. Have you met any occasion that the victim had lost his/her life by your mere absence?
8. In your surroundings do you have any organization to help immediately these victims physically and financially?

For Hospital Staff:

1. What is the procedure that you will do when you hear about an accident victim?
2. Do you give any instructions to ambulance men?
3. Do you assist the attendants of the victim to give first aid requirements?
4. Will you make it mandatory to check for the patients relative and about the money to spend?
5. Have you ever come across any situation where in the life of the victim is lost due to delayed admission?