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GEOMETRIC SERIES IN FINANCIAL MATHEMATICS

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Abstract:

Sequences are fundamental mathematical objects with a long history in mathematics. They are tool for the development of other concepts as well as tools for mathematization of real life situations.

Keywords:

Geometric Sequence & Series, Sigma Notation & Application of Geometric Series

Introduction:

. This paper will cover the study of applications of geometric series in financial mathematics. The geometric series is a marvel of mathematics which rules much of the world. It is finance; however, the geometric series finds perhaps its greatest predictive power.

Notation:

$\{a_n\}$ $\rightarrow$ geometric sequence.
r $\rightarrow$ ratio
n $\rightarrow$ Positive integer

Definition:

A sequence is a particular case of a set. It is denoted by $\{a_n\}$, $n \in N$

Example:

If $a_n = n^2$, n =1,2,3…. we have the sequence 1,4,9,16,…

Definition:

A sequence $\{a_n\}$ is said to be geometric with common ratio r if the terms satisfy the recurrent formula: $a_n = r\, a_{n-1}$. Sometimes, it is called a(GP)geometric progression .Each term in the progression is found by multiplying the previous term.

General form: a, ar, ar^2, ar^3… ar^{n-1}

A geometric sequence is:

Increasing iff $r > 1$
Decreasing iff$0 < r < 1$

Example:

The sequence {1, 3, 9, 27, …} is a geometric sequence with common ratio 3.

Definition:

The sum of several terms of a sequence is called a series.

Definition:

A geometric series is the sum of the elements of a geometric sequence a+ ar+ ar^2+ ar^3+…. + ar^{n-1}

A series can be finite or infinite.

The sum of the first n terms of the geometric series is given by

$$S_n = \begin{cases} na & r=1 \\ \frac{a(1-r^n)}{1-r} & r\neq 1 \end{cases}$$

Definition:

A geometric series, a+ ar+ ar^2+….+ar^{n-1} converges when $|r| < 1$;
i.e $-1 < r < 1$ <. Since if $|r| < 1$, $r^n \rightarrow 0$as $n \rightarrow \infty$ &

$$S_n \rightarrow \frac{a}{1-r} \text{ as } n \rightarrow \infty$$

The limit $\frac{a}{1-r}$ is known as the 'sumass the 'sum to infinity' & is denoted by S_∞.

Sigma Notation to a Geometric Series:

A series can be finite or infinite. In order to reduce the writing of a series, we use the summation symbol .In the case of a geometric series, the terms of the sum are those of a geometric sequence, i.e.$a_n = a_0 r^n$.

Thus, a geometric series of common ratio r has the following form

$$\sum_{n=0}^{N} a_n = \sum_{n=0}^{N} a_0 \ r^n = a_0 + a_1 \ r + a_2 r^2 + \cdots + a_0 \ r^N$$

Example:

Write the following using the summation symbol: $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^7$

Solution:

The terms of the sum are those of the geometric sequence

Here $a_0 = 1$ & $r = 2$ .All terms can be represented by the relation

$a_n = a_0 r^n \to a_n = 1 * 2^n = 2^n$

Thus, the geometric series can be written as follows:

$$\sum_{n=0}^{7} 2^7 = 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^7$$

Formula to Evaluate a Finite Geometric Series:

$$\sum_{n=0}^{N} a_0 \ r^n = a_0 \ \frac{1-r^{N+1}}{1-r},$$

Application of Geometric Series in Financial Mathematics:

- Interest dates : Dates when the interests are deposited;
- Interest period : Time interval between two interest dates;
- Capitalization : adding interests to the capital;
- Periodic interest rate(i): Real interest rate per interest period;
- Nominal interest rate(j): This rate, calculated on an annual basis, is used to determine the periodic rate. It is generally this rate that is posted. It should always be accompanied by a precision on the type of capitalization. Given m number of interest periods in the year d duration of the period in the fraction of a year j nominal rate

Then the periodic rate is then the periodic rate is given by

$$I = \frac{j}{m} = d * j$$

For example, a rate of "8 %biannually capitalized" signifies that the interest period is the half-year (m=2or d = 1/2) and that the periodic rate (biannually) is

$i = \frac{8\%}{2} = 4\%$. The nominal rate does not correspond to the real annual rate, unless the capitalization is annual;

- Effective rate : Real annual interest rate;

In general, if $I_0$is the initial amount invested at the invested at the periodic interest rate "i ", then the, value of the investment after n interest periods V_n, is described by the relation

$$I_n = I_0 (1 + i)^n$$

The sequence of the value of the investment $\{ I_0, I_1, I_2, \ldots \}$ is geometric of common ratio 1 + i.

Example: 1

A student borrows an education loan 25000 $. The bank loans this money at a rate of 9 %, capitalized monthly. What amount will the student have to reimburse two years later?

Solution:

When the interest rate is stated this way, it is the nominal rate. Since the capitalization is monthly, the interest period is one month and the number of periods in the year is m = 12. The periodic rate is then i = 0.09 / 12 = 0,0075per month. The student must reimburse the loan in two years, n = 24 interest periods later.

He needs to reimburse

$I_{24} = I_0 (1+ i)^{24}$

$I_{24} = 25000 (1+ 0.0075)^{24}$

$I_{24} = 299100$

The student will have the amount to reimburse two years later is $ 299100.

Example: 2

Karthick is deposit an amount of $ 2500 on Dec 31 in savings account. The bank gives the rate of 9 %, capitalized monthly. How much will the person have accumulated at the end of the year, immediately after having deposited the last payment the 30 of November?

Solution:

This type of problem, where we need to consider a certain number of equal deposits at regular intervals, can be resolved with geometric series. Instead of making 12 monthly deposits in one account, we could open12 different accounts, as long as the interest rate is the same in each. You would need to deposit anamountI_0in the first account December 31, an amount in the second account January 31, and so forth until the 12thdeposit.

We would accumulate the same amount of money as if we had deposited the 12 payments in the same account. The calculation of the accumulated amount on the 30 of November is done by the sum of the individual acquired values (or cumulated) per deposit. The value of a deposit after n capitalization periods at rate i is obtained from the formula: $I_n = I_0 (1 + i)^n$

From this problem, $I_0 = \$2500$

The periodic rate is then $i = \frac{0.09}{12} = 0.0075$ per month.

S.No	Deposit date	Initial amount	Number of months until January 31	Acquired value of deposit November 30
1	31 Dec	$ 2500	11	2714.16
2	31 Jan	$ 2500	10	2693.96
3	28 Feb	$ 2500	9	2673.90
4	31 Mar	$ 2500	8	2654.00
5	30 Apr	$ 2500	7	2634.24
6	31 May	$ 2500	6	2614.63
7	30 Jun	$ 2500	5	2595.17
8	31 Jul	$ 2500	4	2575.85
9	31 Aug	$ 2500	3	2556.67
10	30 Sep	$ 2500	2	2537.64
11	31 Oct	$ 2500	1	2518.75
12	30 Nov	$ 2500	0	2500.00
			Total	31266.97

The total amount available in the savings account of the person is the sum of the acquired (cumulated) values of each deposit is $31266.97

Using a geometric series of common ratio 0.0075

By applying the general formula:

$$\sum_{n=0}^{N} a_0\, r^n = a_0 \, \frac{1-r^{N+1}}{1-r}$$

Here $a_0 = I_0$, N = 12 & r = (1 + i) we have,

$$M = I_0\left[\frac{(1+i)^{12}-1}{i}\right]$$

Here I_0 = 2500, i = 0, 0075

$$= 2500\left[\frac{(1.0075)^{12}-1}{0.0075}\right]$$

$$M = 31266.97$$

Therefore Karthick will get the money at the end of the year is $ 31266.97

Conclusion:

Many real world applications of sequences & series have the feature that data is collected consecutively in time. A few examples of the stock market & bank investment. We refer to any collection lie this of observations as a sequences & series. This paper provided by the geometric series & how it can be used to solve certain application.

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